

Quadratic Curve of Best Fit

Reporting Category Statistics

Topic Determining a quadratic curve of best fit

Primary SOL A.11 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems, using mathematical models. Mathematical models will include linear and quadratic functions.

Related SOL A.4, A.7

Materials

- Straight edge and compass (optional)
- Graph paper
- Graphing calculators (optional)

Vocabulary

circle, chord (earlier grades)

independent variable, dependent variable (A.7)

curve of best fit (A.11)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Have each student draw a circle and put one point on it. Ask how many chords can be drawn using this one point. (0) Have students put a second point on their circles. Ask how many chords can be drawn now. (1) Have them put a third point on their circles, and ask how many chords can be drawn now. (3) Continue this process up to 9 points on the circle. Record this data in the *first two columns* of a display table like the one at right.
2. Ask students to identify the independent and dependent variables in this situation. Distribute graph paper, and have students graph these data points with the independent variable values (number of points) on the x-axis and the dependent variable values (number of chords) on the y-axis.
3. Discuss what shape these data points make, and lead students to discover that this is not a linear relationship.
4. Have students draw a curve of best fit by connecting any three data points that seem to best represent the data.
5. Direct students to use this curve to make the following predictions:
 - How many chords can be drawn with 10 points? (45)
 - How many points are used to draw 105 chords? (15)

Number of Points	Number of Chords	First Change	Second Change
1	0		
2	1	1	
3	3	2	1
4	6	3	1
5	10	4	1
6	15	5	1
7	21	6	1
8	28	7	1
9	36	8	1

6. Next, fill in column three in the table by showing students that progressing from 0 to 1 chord is 1, from 1 to 3 chords is 2, from 3 to 6 chords is 3, from 6 to 10 chords is 4, etc. Then, demonstrate filling in column four. Point out that the second change of the output is constant, and inform students that this is a characteristic of a quadratic equation.

7. Have students write the equation for the data collected. There are several ways to determine the quadratic equation. The easiest method is to enter the data in the graphing calculator and use the quadratic regression function to get $y = 0.5x^2 - 0.5x$. You can also discover the equation through class discussion, as follows: Post,

“Number of chords = Multiplier \times Number of points”

Also, display a data chart like the one at right. Have students help you fill in the multipliers in the chart. Then, have students write all the multipliers as

fractions with a common denominator: $\frac{0}{2}, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2}, \frac{7}{2}, \frac{8}{2}$. Point out that

the numerator of each multiplier is one less than the number of points.

Letting x represent the number of points and y represent the number of

chords, we get the equation $y = \frac{(x-1)x}{2}$ or $y = 0.5x^2 - 0.5x$.

8. Have students predict how many chords can be drawn with 100 points on the circle. (4,950) Then, have them use the equation to calculate the number of chords with 100 points and compare the solution to their predictions. If there is a discrepancy, ask why, and have students explain whether it is significant or not.

Number of chords	Multiplier	Number of points
0 =	$0 \times$	1
1 =	$\frac{1}{2} \times$	2
3 =	$1 \times$	3
6 =	$\frac{3}{2} \times$	4
10 =	$2 \times$	5
15 =	$\frac{5}{2} \times$	6
21 =	$3 \times$	7
28 =	$\frac{7}{2} \times$	8
36 =	$4 \times$	9

Assessment

- **Questions**
 - What is a curve of best fit?
 - Compare a linear curve of best fit (line of best fit) to a quadratic curve of best fit. What is the difference in the data? How does this difference affect the equations?
- **Journal/Writing Prompts**
 - Describe a real-world situation in which the collected data would lend itself to a quadratic relationship.
 - Describe the difference between making predictions from a graph and making predictions from an equation. When is a graph most helpful? When is an equation most helpful?

Extensions and Connections (for all students)

- Ask students to brainstorm ideas of real-world situations that may encompass quadratic relationships. Have them use the Internet to collect the data, graph it, and find the quadratic equation for it.

Strategies for Differentiation

- Provide predrawn circles for students who need them.
- Have students use a straight edge to facilitate making straight lines.
- Demonstrate by filling in the first few rows of each table.